MAT 1348 3X – Practice Test # 2 – Spring/Summer 2016

Name:________________________________

Student Number:________________________________

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Question	Possible Points	Points Obtained
# 1	5	
# 2	5	
# 3	5	
# 4	5	
# 5	5	
Total	30	

Instructions:

- Print your name and student number on the first two pages.
- Verify that your copy of the test has all of its 8 pages.
- You must answer all questions. There are 5 questions for a total of 25 points.
- Write the solutions to the questions in the space provided. You may use the back of the pages if necessary.

SHOW ALL YOUR WORK

1. (**5 points**) Identify the proof method used to demonstrate the following assertions.

   (a) "Let n be a positive integer. If n^2 is odd, then n is odd."

   Suppose that n is even, then we can write $n = 2m$ with m some integer; we deduce that $n^2 = 4m^2 = 2(2m^2)$, which is thus also even.

   (b) "If x and y are odd integers, then their product xy is odd."

   First, note that x is an odd integer, thus we can write $x = 2m + 1$ where m is some integer. Similarly, $y = 2n + 1$ where n is some integer. Consequently, $xy = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$, which indeed gives rise to an odd integer.

   (c) "Let x andy be two real numbers. Prove that $\max\{x, y\} + \min\{x, y\} = x + y$."

   Let x and y be two real numbers, then we consider two possible cases:

   Case 1: If $x \le y$,
$$\max\{x, y\} + \min\{x, y\} = y + x = x + y.$$
   Case 2: If $x > y$,
$$\max\{x, y\} + \min\{x, y\} = x + y.$$
   In both cases we have that $\max\{x, y\} + \min\{x, y\} = x + y$. Hence, this assertion is true.

(d) "If $x^2 = 2$, then x is not a rational number."

Suppose that x is a fraction and write $x = m/n$ with m and n integers, with $n \neq 0$. Moreover, suppose that the integers m and n do not have any common divisors.
Given that $x^2 = 2, (m/n)^2 = 2$. Consequently, $m^2 = 2n^2$.
But this implies that m^2 is an even integer. It follows that m is then an even integer. Thus we can write $m = 2p$ with p come integer. Substituting m with $2p$ in the equation $m^2 = 2n^2$, we obtain $n^2 = 2p^2$. But then, n^2 is an even number, and thus n is also even. We have thus shown that m and n are both even, which leads us to conclude that they have a common divisor (2). This conclusion contradicts our initial proposition stating that m and n did not have any common divisors. Therfore, if $x^2 = 2$, then x is not a fraction.

(e) "The sum $1 + 2 + \ldots + n = n(n+1)/2$ for all integers $n \geq 1$."

For $n = 1$, the left-hand side of the equation is simply 1 and the right-hand side is $(1)(1+1)/2 = 1$. Consequently, the proposition $P(1)$ is true.
Now suppose that $1 + 2 + \ldots + k = k(k+1)$ for some integer $k \geq 1$. Therefore, $1+2+\ldots+(k+1) = (1+2+\ldots+k)+(k+1) = k(k+1)/2 + (k+1) = [k(k+1)+2(k+1)]/2 = [(k+2)(k+1)]/2 = (k+1)(k+2)/2$. Hence, our assertion is true for any integer $n \geq 1$.

2. (**5 points**) Use an indirect proof to prove that if x is irrational and $x \geq 0$, then $\sqrt{x}$ is irrational.

3. (**5 points**) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is as a sum of a subset of the integers $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and so on.

4. [**5 points**]

(a) (**1 point**) Determine if each statement is true or false.

(i) $\{x\} \subseteq \{x\}$.

(ii) $\emptyset \in \{x\}$.

(b) (**1 point**) What is the cardinality of each of the following sets?

(i) $P(\{\emptyset, a, \{a, b\}\})$

(ii) $\{a, b, c, d\} \times \{x, y, z\}$

(c) (**3 points**) Prove that

$$(B - A) \cap (C - A) = (B \cap C) - A$$

5. (**5 points**)

(a) (**3 points**) Given the following sets:

$$S = \{b, c, d, e\},\ T = \{a, d\} \text{ and } R = \{b, c\}.$$

Find

(i) $P(T)$

(ii) $T \times R$

(iii) $(S \cap T) \cup R$

(b) (**2 points**) If U is the universal set $U = \{a, b, c, d, e, f\}$, draw a Venn diagram depicting the sets S, T, and R. Please make sure your diagram shows each element of the universal set as a point.